Ex:- find the equation of the surface generated by the Revolution of the circle

$$x^2+y^2-2ay+a^2-r^2=0,\ z=0$$ about the X-axis $(a>r)$

→ The equations of the curve are -

$$x^2+y^2-2ay+a^2-r^2=0,\ z=0$$

∴ The curve lies in the plane $z=0$ i.e in the xy plane

The equation of curve in xy plane is

$$x^2+y^2-2ay+a^2-r^2=0 \quad \text{——} \ (1)$$

Let $P(x,y)$ be any point on the curve in the xy-plane

As the curve revolves about the X-axis -

(i) The point $P(x,y)$ in the xy plane becomes the point $P(x,y,z)$ in space.

(ii) The $\perp$ distance of $P(x,y)$ from the axis of Revolution, the X-axis, $= y$ becomes the $\perp$ distance of $P(x,y,z)$ from the X-axis $= \sqrt{y^2+z^2}$

$\therefore$ from ①, changing y to $\sqrt{y^2+z^2}$ we get

$$x^2+y^2+z^2-2a\sqrt{y^2+z^2}+a^2-r^2=0$$

$$x^2+y^2+z^2+a^2-r^2=2a\sqrt{y^2+z^2}$$

Sq. b.s

$$(x^2+y^2+z^2+a^2-r^2)^2=4a^2(y^2+z^2)$$

which is required equation of surface of Revolution.

Ex- Identify the surface

$$4x^2 + 9y^2 + z^2 - 6x + 6y - 4z + 10 = 0 \quad (i)$$

$$\rightarrow \quad (4x^2 - 6x) + (9y^2 + 6y) + (z^2 - 4z) + 10 = 0 \quad (ii)$$

$$4\left(x^2 - \frac{3}{2}x\right) + 9\left(y^2 + \frac{2}{3}y\right) + (z^2 - 4z) + 10 = 0$$

Completing the squares,

$$4\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) + 9\left(y^2 + \frac{2}{3}y + \frac{1}{9}\right) + (z^2 - 4z + 4) + 10 - \frac{9}{4} - 1 - 4 = 0$$

$$4\left(x - \frac{3}{4}\right)^2 + 9\left(y + \frac{1}{3}\right)^2 + (z-2)^2 + 10 - \frac{9}{4} - 5 = 0$$

$$4\left(x - \frac{3}{4}\right)^2 + 9\left(y + \frac{1}{3}\right)^2 + (z-2)^2 + \frac{11}{4} = 0$$

Shifting the origin to $\left(\frac{3}{4}, \frac{1}{3}, 2\right)$ the above equation reduces to $4X^2 + 9Y^2 + Z^2 + \frac{11}{4} = 0$

which represents an empty set

Ex- Identify the surface $x^2+4y^2+9z^2+4x+12y-6z+14=0$

→ The given surface is

$$x^2+4y^2+9z^2+4x+12y-6z+14=0$$

$$(x^2+4x)+(4y^2+12y)+(9z^2-6z)+14=0$$

$$(x^2+4x)+4\left(y^2+3y\right)+9\left(z^2-\frac{2}{3}z\right)+14=0$$

Completing the squares.

$$(x^2+4x+4)^2+4\left(y^2+3y+\frac{9}{4}\right)+9\left(z^2-\frac{2}{3}z+\frac{1}{9}\right)$$

$$+14-4-9-1=0$$

$$(x+2)^2+4\left(y+\frac{3}{2}\right)^2+9\left(z-\frac{1}{3}\right)^2+14-14=0$$

$$(x+2)^2+4\left(y+\frac{3}{2}\right)^2+9\left(z-\frac{1}{3}\right)^2=0$$

Shifting the origin to $\left(-2,-\frac{3}{2},\frac{1}{3}\right)$ the above equation reduces to

$$X^2+Y^2+9Z^2=0$$ which represents single point

Ex- Reduce the following equation to the Standard form and Identify the curve

$$x^2 - 4xy + 4y^2 - 32x + 4y + 16 = 0$$

→ The given equation is

$$x^2 - 4xy + 4y^2 - 32x + 4y + 16 = 0$$

To remove the term of xy we are to rotate axes through θ

$$\tan 2\theta = \frac{2h}{a-b} = \frac{-4}{1-4} = \frac{-4}{-3} = \frac{4}{3}$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{4}{3}$$

$$6\tan\theta = 4 - 4\tan^2\theta$$

$$2(3\tan\theta) = 2(2 - 2\tan^2\theta)$$

$$3\tan\theta = 2 - 2\tan^2\theta$$

$$2\tan^2\theta + 3\tan\theta - 2 = 0$$

$$\tan\theta = \frac{-3 \pm \sqrt{9 - 4(2)(-2)}}{4} = \frac{-3 \pm \sqrt{9+16}}{4}$$

$$= \frac{-3 \pm \sqrt{25}}{4}$$

$$= \frac{-3 \pm 5}{4}$$

$$= \frac{-3+5}{4}, \quad \frac{-3-5}{4}$$

$$= \frac{2}{4}, \quad \frac{-8}{4}$$

$$= \frac{1}{2}, \quad -2$$

Take $\tan\theta = \frac{1}{2} = \frac{P}{B}$

Since $P = 1, \quad B = 2$

$$H^2 = P^2 + B^2$$

$$= 1 + 4$$

$$H^2 = 5$$

$$H = \sqrt{5}$$

$$\sin\theta = \frac{P}{H} = \frac{1}{\sqrt{5}} \qquad \cos\theta = \frac{B}{H} = \frac{2}{\sqrt{5}}$$

$$x = X\cos\theta - Y\sin\theta = \frac{2X}{\sqrt{5}} - \frac{Y}{\sqrt{5}} = \frac{2X-Y}{\sqrt{5}}$$

$$y = X \sin\theta + Y \cos\theta$$

$$= \frac{X}{\sqrt{5}} + \frac{2Y}{\sqrt{5}}$$

$$= \frac{X + 2Y}{\sqrt{5}}$$

The Transformed equation is

$$\frac{(2X-Y)^2}{\sqrt{5}} - 4\,\frac{(2X-Y)}{\sqrt{5}}\left(\frac{X+2Y}{\sqrt{5}}\right) + 4\,\frac{(X+2Y)^2}{(\sqrt{5})^2}$$

$$-32\left(\frac{2X-Y}{\sqrt{5}}\right) + 4\left(\frac{X+2Y}{\sqrt{5}}\right) + 16 = 0$$

$$4X^2 + Y^2 - 4XY - 4(2X^2 - 2Y^2 + 3XY) + 4(X^2 + 4Y^2 + 4XY)$$

$$-32\sqrt{5}(2X-Y) + 4\sqrt{5}(X+2Y) + 80 = 0$$

$$25Y^2 - 60\sqrt{5}\,X + 40\sqrt{5}\,Y + 80 = 0$$

$$5Y^2 - 12\sqrt{5}\,X + 8\sqrt{5}\,Y + 16 = 0$$

which is parabolic cylinder